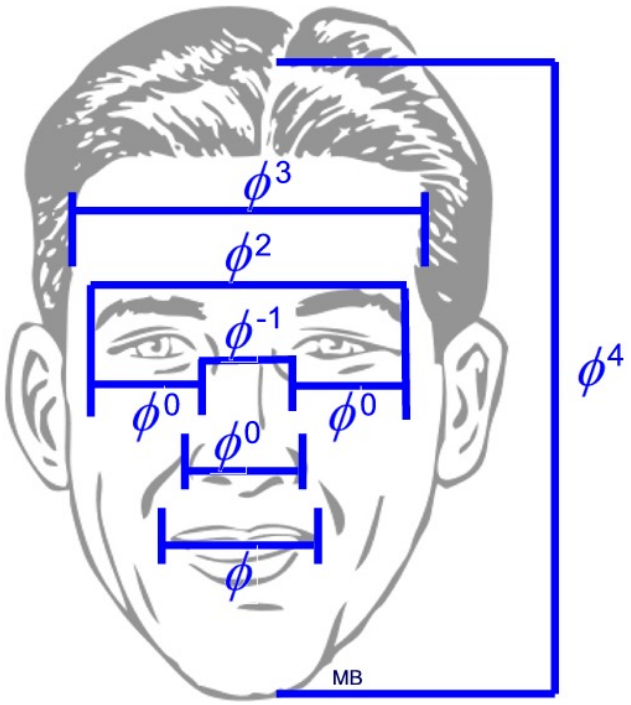
Fibonacci Faces

Name_____

The Beautiful People

Need to know: The Golden Ratio, Phi (ϕ pronounced "fi") = 1.6180339891.



1. The facial mask at the left contains references to ϕ raised to various powers. Compute approximate values for these powers:

ϕ^4 = _____

ϕ^3 = _____

ϕ^2 = _____

ϕ = _____

ϕ^0 = _____

ϕ^{-1} = _____

2. Fibonacci Sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, _____, _____, _____, _____, _____, ...
The ratio of successive pairs of this sequence tends to a famous number called the **golden section,** 1.618033989. The ratio 1.6180339891 **:** 1 is called the **golden ratio, Phi.** Use your graphing calculator to examine the tendency of the Fibonacci Sequence to approach the golden section. Enter the numbers from 1 to 12 in List 1 and the accompanying ratios in List 2. Produce a scatter plot. Enter Y1 = 1.618033989. Graph and examine.

1. 2/1
2. 3/2
3. 5/3
4. 8/5
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____

Describe explicitly how the Fibonacci ratios relate to Phi. Be sure to describe "how" the values are approaching 1.618033989. _____
_____
_____
_____
_____

3. Phi, the golden ratio of 1.618033989, can be derived mathematically by solving the equation:

$$n^2 - n - 1 = 0.$$

Show that this equation can be solved algebraically to produce the value of Phi. Express your answer in fractional form. Do not use your calculator for a decimal solution to this equation. Use the calculator only to check.

4. Phi can be used to create some interesting equations. Which of the following statements relating to Phi are true?

a.) $\phi^2 = \phi + 1$ _____

b.) $1 / \phi = \phi - 1$ _____

c.) $\phi^3 = \phi^2 + \phi$ _____

d.) $\phi = 5^{0.5} * 0.5 + 0.5$ _____

5. Phi can be used to compute any number of the Fibonacci Sequence, $f(n)$, by using the formula:

$$f(n) = \phi^n / \sqrt{5}$$

This formula will produce an estimate that will round to the correct Fibonacci number.

Using this formula, compute the indicated Fibonacci value:

a.) 20th term _____

b.) 35th term _____

Computing a more exact value needs a more intense formula: $f(n) = [\phi^n - (-\phi)^{-n}]/(2\phi - 1)$

Using this formula, compute the indicated Fibonacci value:

c.) 15th term _____

d.) 40th term _____