## Sequences and Series:

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Binomial Theorem</strong></td>
<td></td>
</tr>
<tr>
<td>((a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k)</td>
<td></td>
</tr>
<tr>
<td><strong>Arithmetic Last Term</strong></td>
<td>(a_n = a_1 + (n-1)d)</td>
</tr>
<tr>
<td><strong>Geometric Last Term</strong></td>
<td>(a_n = a_1 r^{n-1})</td>
</tr>
<tr>
<td><strong>Find the}^{th} Term</strong></td>
<td>(\left(\begin{array}{c}n \ r-1\end{array}\right) a^{n-(r-1)} b^{r-1})</td>
</tr>
<tr>
<td><strong>Arithmetic Partial Sum</strong></td>
<td>(S_n = n \left(\frac{a_1 + a_n}{2}\right))</td>
</tr>
<tr>
<td><strong>Geometric Partial Sum</strong></td>
<td>(S_n = a_1 \left(\frac{1 - r^n}{1 - r}\right))</td>
</tr>
</tbody>
</table>

### Functions:

**To find the inverse function:**

1. Set function = \(y\)
2. Interchange the variables
3. Solve for \(y\)

**Composition of functions:**

\((f \circ g)(x) = f(g(x))\)
\((g \circ f)(x) = g(f(x))\)
\((f \circ f^{-1})(x) = x\)

**Algebra of functions:**

\((f + g)(x) = f(x) + g(x)\);
\((f - g)(x) = f(x) - g(x)\)
\((f \cdot g)(x) = f(x) \cdot g(x)\);
\((f / g)(x) = f(x) / g(x)\), \(g(x) \neq 0\)

**Domains:** \(D(f(x)) \cap D(g(x))\)

### Asymptotes (vertical)

- Check to see if the denominator could ever be zero.
- \(f(x) = \frac{x}{x^2 + x - 6}\)
- Vertical asymptotes at \(x = -3\) and \(x = 2\)

### Asymptotes (horizontal)

1. \(f(x) = \frac{x+3}{x^2 - 2}\)
   - top power < bottom power means \(y = 0\) (z-axis)
2. \(f(x) = \frac{4x^2 - 5}{3x^2 + 4x + 6}\)
   - top power = bottom power means \(y = 4/3\)
   - (coefficients)
3. \(f(x) = \frac{x^3}{x+4}\)
   - None!
   - top power > bottom power

### Complex and Polars:

\[ [r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta) \]
\[ r = \sqrt{a^2 + b^2} \quad x = r \cos \theta \]
\[ \theta = \arctan \frac{b}{a} \quad y = r \sin \theta \]
\[ a + bi \quad i = \sqrt{-1} \]

### Determinants:

\[ \begin{vmatrix} 3 & 5 \\ 4 & 3 \end{vmatrix} = 3 \cdot 3 - 5 \cdot 4 \]

**Cramer’s Rule:**

\[ \begin{vmatrix} a & b & c \\ d & e & f \end{vmatrix} = \frac{1}{\left| \begin{vmatrix} c & b \\ d & e \end{vmatrix} \right|} \]

- Also apply Cramer’s rule to 3 equations with 3 unknowns

### Trig:

**Reference Triangles:**

- \(\sin \theta = \frac{o}{h}\); \(\cos \theta = \frac{a}{h}\); \(\tan \theta = \frac{o}{a}\)
- \(\csc \theta = \frac{h}{o}\); \(\sec \theta = \frac{h}{a}\); \(\cot \theta = \frac{a}{o}\)

**BowTie**
## Analytic Geometry:

### Circle

\[(x - h)^2 + (y - k)^2 = r^2\]

Remember “completing the square” process for all conics.

### Ellipse

\[\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1\]

- Larger denominator → major axis and smaller denominator → minor axis
- \(c\) → focus length where major length is hypotenuse of right triangle.
- Latus rectum lengths from focus are \(\frac{b^2}{a}\)

### Hyperbola

\[\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1\]

- \(a\)→transverse axis
- \(b\)→conjugate axis
- \(c\)→focus where \(c\) is the hypotenuse.
- Asymptotes needed

### Parabola

\[(x - h)^2 = 4a(y - k)\]

- Vertex to focus = \(a\), length to directrix = \(a\), latus rectum length from focus = \(2a\)

### Hyperbola

\[\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1\]

- Latus length from focus \(\frac{b^2}{a}\)

### Eccentricity:

- \(e = 0\) circle
- \(0 < e < 1\) ellipse
- \(e = 1\) parabola
- \(e > 1\) hyperbola

### Induction:

- Find \(P(1)\):
- Assume \(P(k)\) is true:
- Show \(P(k+1)\) is true:

## Polynomials:

### Remainder Theorem:

Substitute into the expression to find the remainder.

\([x + 3]\) substitutes -3

### Synthetic Division

Mantra:

“Bring down, multiply and add, multiply and add…”

[when dividing by \((x - 5)\), use +5 for synthetic division]

### Depress equation

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

(Also use calculator to examine roots)

### Analysis of Roots

P N C Chart

* All rows add to the degree
* Complex roots come in conjugate pairs
* Product of roots - sign of constant (same if degree even, opposite if degree odd)
* Decrease P or N entries by 2

### Descartes’ Rule of Signs

1. Maximum possible # of positive roots → number of sign changes in \(f(x)\)
2. Maximum possible # of negative roots → number of sign changes in \(f(-x)\)

### Far-left/Far-right Behavior of a Polynomial

The leading term \((a_nx^n)\) of the polynomial determines the far-left/far-right behavior of the graph according to the following chart. (“Parity” of \(n \rightarrow\) whether \(n\) is odd or even.)

<table>
<thead>
<tr>
<th>(a_nx^n)</th>
<th>LEFT-HAND BEHAVIOR (same as right)</th>
<th>LEFT-HAND BEHAVIOR (opposite right)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n) is even</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(n) is odd</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Rate of Growth/Decay:

\[y = y_0e^{kt}\]

\(y\) = end result, \(y_0\) = start amount,

Be sure to find the value of \(k\) first.