PreCalculus Formulas

Sequences and Series:

Binomial Theorem	Arithmetic Last Term	Geometric Last Term
$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$	$a_n = a_1 + (n-1)d$	$a_n = a_1 r^{n-1}$
Find the r^{th} term	**Arithmetic Partial Sum**	**Geometric Partial Sum**
$\binom{n}{r-1} a^{n-(r-1)} b^{r-1}$	$S_n = n\left(\dfrac{a_1 + a_n}{2}\right)$	$S_n = a_1\left(\dfrac{1-r^n}{1-r}\right)$

Complex and Polars:

DeMoivre's Theorem:

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\cdot\theta + i\sin n\cdot\theta)$$

$r = \sqrt{a^2 + b^2}$ $\quad x = r\cos\theta$

$\theta = \arctan\dfrac{b}{a}$ $\quad y = r\sin\theta$

$(r,\theta) \rightarrow (x,y)$

$a + bi$
$i = \sqrt{-1}$
$i^2 = -1$

Functions:

To find the inverse function: $f^{-1}(x)$

1. Set function $= y$
2. Interchange the variables
3. Solve for y

Composition of functions:

$(f \circ g)(x) = f(g(x))$

$(g \circ f)(x) = g(f(x))$

$(f \circ f^{-1})(x) = x$

Algebra of functions: $(f+g)(x) = f(x) + g(x)$; $\quad (f-g)(x) = f(x) - g(x)$

$(f \cdot g)(x) = f(x) \cdot g(x)$; $\quad (f/g)(x) = f(x)/g(x),\ g(x) \neq 0$

Domains:: $D(f(x)) \cap D(g(x))$

Domain (usable x's)
Watch for problems with zero denominators and with negatives under radicals.

Range (y's used)

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

terms not containing a mult. of h will be eliminated.

Asymptotes: (vertical)
Check to see if the denominator could ever be zero.

$$f(x) = \frac{x}{x^2 + x - 6}$$

Vertical asymptotes at $x = -3$ and $x = 2$

Asymptotes: (horizontal)

1. $f(x) = \dfrac{x+3}{x^2 - 2}$

top power < bottom power means y = 0 (z-axis)

2. $f(x) = \dfrac{4x^2 - 5}{3x^2 + 4x + 6}$

top power = bottom power means y = 4/3 (coefficients)

3. $f(x) = \dfrac{x^3}{x+4}$ None!

top power > bottom power

Determinants:

$$\begin{vmatrix} 3 & 5 \\ 4 & 3 \end{vmatrix} = 3\cdot 3 - 5\cdot 4$$

Use your calculator for 3x3 determinants.

Cramer's Rule:

$ax + by = c$

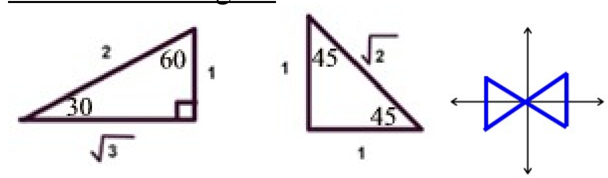
$dx + ey = f$

$$\frac{1}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}\left(\begin{vmatrix} c & b \\ f & e \end{vmatrix}, \begin{vmatrix} a & c \\ d & f \end{vmatrix}\right)$$

Also apply Cramer's rule to 3 equations with 3 unknowns.

Trig:

Reference Triangles:



$\sin\theta = \dfrac{o}{h}$; $\quad \cos\theta = \dfrac{a}{h}$; $\quad \tan\theta = \dfrac{o}{a}$ $\quad$ BowTie

$\csc\theta = \dfrac{h}{o}$; $\quad \sec\theta = \dfrac{h}{a}$; $\quad \cot\theta = \dfrac{a}{o}$

Analytic Geometry:

Circle

$(x-h)^2+(y-k)^2=r^2$

Remember "completing the square" process for all conics.

Ellipse

$\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$

larger denominator → major axis and smaller denominator → minor axis

c → focus length where major length is hypotenuse of right triangle.

Latus rectum lengths from focus are b^2/a

Eccentricity:

$e=0$ circle
$0<e<1$ ellipse
$e=1$ parabola
$e>1$ hyperbola

Parabola

$(x-h)^2=4a(y-k)$

$(y-k)^2=4a(x-h)$

vertex to focus = a, length to directrix = a, latus rectum length from focus = 2a

Hyperbola

$\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$

Latus length from focus b^2/a

a→transverse axis
b→conjugate axis
c→focus where c is the hypotenuse.
asymptotes needed

Induction:

Find P(1):
Assume P(k) is true:
Show P(k+1) is true:

Rate of Growth/Decay: $y=y_0e^{kt}$

y = end result, y_0 = start amount,
Be sure to find the value of *k* first.

Polynomials:

Remainder Theorem:
Substitute into the expression to find the remainder.
[(*x* + 3) substitutes -3]

Synthetic Division Mantra:
"Bring down, multiply and add, multiply and add…"
[when dividing by (*x* - 5), use +5 for synthetic division]

Depress equation

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

(also use calculator to examine roots)

Descartes' Rule of Signs

1. Maximum possible # of positive roots → number of sign changes in $f(x)$

2. Maximum possible # of negative roots → number of sign changes in $f(-x)$

Analysis of Roots

P N C Chart
* all rows add to the degree
* complex roots come in conjugate pairs
* product of roots - sign of constant (same if degree even, opposite if degree odd)
* decrease P or N entries by 2

Upper bounds:
All values in chart are +

Lower bounds:
Values alternate signs

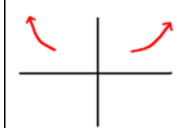
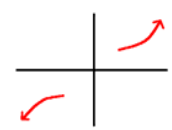
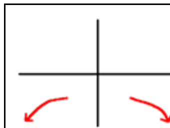
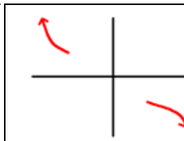
No remainder: Root

Sum of roots is the coefficient of second term with sign changed.

Product of roots is the constant term (sign changed if odd degree, unchanged if even degree).

Far-left/Far-right Behavior of a Polynomial

The leading term (a_nx^n) of the polynomial determines the far-left/far-right behavior of the graph according to the following chart. ("Parity" of *n* → whether *n* is odd or even.)

<table>
<tr><td rowspan="2" colspan="2">a_nx^n</td><td colspan="2">LEFT-HAND BEHAVIOR</td></tr>
<tr><td>n is even
(same as right)</td><td>n is odd
(opposite right)</td></tr>
<tr><td rowspan="2">RIGHT-HAND BEHAVIOR or
Leading Coefficient Test</td><td>$a_n>0$</td><td>
always positive</td><td>
negative $x<0$
positive $x>0$</td></tr>
<tr><td>$a_n<0$</td><td>
always negative</td><td>
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